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Investigating Critical Thinking Indicators in the Context of Algebra Problem Solving: A Study in Indonesia

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ABSTRACT

Developing students' critical thinking in mathematics, particularly in algebra problem solving, is crucial for enhancing conceptual understanding and mathematical reasoning. Although critical thinking has been widely studied, limited research has specifically examined how Indonesian students demonstrate critical thinking in algebra tasks, especially in the evaluation stage. This study investigates how students engage with four key indicators of critical thinking: Analysis, Strategy, Inference, and Evaluation, during algebra problem solving. A qualitative descriptive approach was employed, involving six purposively selected eleventh-grade students from Senior High School 5 Tegal, categorized into high, medium, and low levels of critical thinking ability. Data were gathered through classroom tests, structured observations, and in-depth interviews, then analyzed using thematic coding with triangulation to ensure credibility. The results indicate that students with high critical thinking levels were able to perform all four indicators effectively, while those at the medium level showed inconsistent performance, particularly in the inference and evaluation stages. Students with low critical thinking levels struggled to identify mathematical structures, often applying procedures without conceptual clarity. These findings suggest that the quality of students' critical thinking is closely related to their algebraic understanding and ability to apply reasoning strategies. This study highlights the underexplored evaluation dimension of critical thinking, offering new insights into how students validate and justify solutions in mathematics. The findings underscore the need for instructional practices that support reflective thinking, encourage reasoning, and promote deeper engagement with mathematical problems. Further research is recommended to explore how these skills develop across time and educational contexts.

1. Introduction

In today's fast-evolving educational landscape, fostering students' ability to think critically has become not only a pedagogical priority but a societal necessity. The rapid growth of knowledge and the complexity of global challenges demand that students be equipped with cognitive tools that enable them to question assumptions, analyze information systematically, and make reasoned judgments. Mathematics, as a foundational subject, provides a fertile ground for developing such critical faculties, particularly through problem solving. The ability to apply mathematical concepts logically and flexibly in diverse situations is widely recognized as essential for nurturing 21st-century competencies (NTCM, 2000; Kaya & Aydin, 2016).

Algebra is a key area in mathematics where students build conceptual understanding and analytical reasoning. Critical thinking is essential in this process, as it enables learners to define concepts, build logical arguments, assess solutions, and provide well-founded justifications for their conclusions. (Chasanah et al., 2020; Dewi Yuarna et al., 2020; Jumaisyaroh & Hasratuddin, 2016). When students engage in algebraic tasks, such as symbolic manipulation or interpreting expressions, they encounter opportunities to transform abstract ideas into meaningful representations. These experiences, when supported by reflective dialogue and structured scaffolding, contribute to a more sophisticated grasp of mathematical relationships (Aprilianingrum & Wardani, 2021; Viseu & Oliveira, 2012; Wilkinson et al., 2018).

Scholars have long emphasized the importance of critical thinking indicators in mathematics, including Analysis, Strategy, Inference, and Evaluation (Sumarmo, 2006; Hendriana & Kadarisma, 2019). Numerous studies have explored how students' written explanations, representational fluency, and verbal justifications serve as indicators of mathematical reasoning (Ahdhianto et al., 2020; Anggraini & Fauzan, 2018; Setiyani et al., 2020). Moreover, it is evident from empirical research that mathematical communication, justification, and problem solving can be significantly enhanced through instructional models that prioritize cognitive engagement (Kusumah et al., 2020; Zakiri et al., 2018; Pourdavood et al., 2020). Yet, there remains a tendency among students to emphasize correctness of answers over the rationale behind their procedures (D. Kurniawan et al., 2017). This behavioral pattern diminishes opportunities for developing reflective and evaluative thinking.

A growing body of literature from diverse cultural and educational contexts has sought to map how students navigate mathematical tasks using various forms of reasoning (Madden et al., 2018; Borg, 2014; Liu, 2020). While some frameworks, such as those proposed by Facione or Ennis, provide a general view of critical thinking, few studies have operationalized these constructs specifically within the Indonesian mathematics curriculum. Additionally, although studies have examined the integration of technology (Kaya & Aydin, 2016) or mental computation (Pourdavood et al., 2020) in promoting critical thinking, they often overlook the micro-cognitive processes students experience when solving algebra problems. Research by Anjarwati et al. (2022) and Shannon and Austin (1992) underlines the value of open-ended tasks and discovery-based models but lacks a granular focus on how students validate, adapt, and critique mathematical ideas within the evaluation phase. Despite emerging interest in linking critical thinking to problem-solving frameworks, the evaluation dimension continues to be underexplored (Ambarwati & Kurniasih, 2021; Jun-On et al., 2022; Shannon & Austin, 1992).

It is in this gap that the present study situates itself. While existing models address broad indicators of critical thinking, this study uniquely highlights the evaluation indicator as a critical yet often neglected component of students' mathematical reasoning. Evaluation, in this context, refers not simply to checking for correctness but to validating the logical coherence of mathematical arguments, verifying assumptions, and examining the fit between the method and the result. This focus is particularly timely given the need for instructional approaches that go beyond rote procedures and encourage meta-cognitive reflection. By observing how Indonesian high school students reason through derivative problems, this study offers a distinctive contribution to understanding the dynamics of critical thinking in an educational setting that is both culturally and pedagogically specific.

This study is significant in several respects. It aims to analyze how students with varying levels of critical thinking ability engage with algebra problems, specifically through the four indicators: Analysis, Strategy, Inference, and Evaluation. Using qualitative methods including problem-solving tests and in-depth interviews, the study investigates how students construct and articulate mathematical understanding in real-time. The objective is not only to describe their performances but also to uncover patterns of reasoning and missteps that reveal deeper insights into their cognitive development. This research thus serves both as a diagnostic and developmental lens for educators and curriculum designers.

The structure of the paper follows the standard IMRAD framework. After this introduction, the next section describes the methodology used to select participants, develop instruments, and analyze qualitative data through coding and triangulation techniques. The findings section presents student performance across all critical thinking indicators with illustrative examples. The discussion then links these findings to existing theories and international literature, providing pedagogical and theoretical implications. The conclusion synthesizes the results and proposes recommendations for practice and future research.

In sum, this study contributes to the literature by centering the evaluation phase as a key indicator of critical thinking in mathematics learning. It reveals how students' struggles or successes in justifying their solutions are deeply connected to conceptual understanding and metacognitive awareness. The findings suggest that instructional strategies in Indonesia must be refined to include evaluative tasks that challenge students to defend, critique, and reflect on their mathematical reasoning. As educational reform in Indonesia increasingly emphasizes higher-order thinking, this research provides timely evidence to inform policy, curriculum, and teacher professional development.

2. Methods

2.1. Research Design and Data Collection

This study employed a qualitative descriptive approach to explore how students demonstrated the indicators of critical thinking in solving algebra problems involving derivatives. This design allowed the researcher to examine students' reasoning processes in depth within their natural learning environment (Creswell, 2013).

The study was conducted at Senior High School 5 Tegal, Central Java, during the 2024 to 2025 academic year. Thirty-five eleventh-grade students participated in the preliminary testing phase. From this group, six students were purposively selected based on their Critical Thinking Ability Test scores, classified as high (63 to 100), medium (47 to 62), and low (below 47).

Data were collected using three instruments. A mathematics test on derivative problems assessed students' conceptual understanding and critical thinking. Semi-structured interviews explored their reasoning in depth, while classroom observations captured real-time engagement. All instruments were reviewed by two mathematics education experts and a senior high school teacher. A pilot study was conducted to ensure clarity and suitability.

The conceptual framework for this study used four core indicators of critical thinking: Analysis, Strategy, Inference, and Evaluation. These indicators are presented in Table 2.1 and served as the coding foundation during analysis.

Table 2.1 Criteria of Critical Thinking Criteria

Criteria	Score
High	63 - 100
Medium	47 - 62
Low	< 47

All interviews were audio recorded and transcribed verbatim. Student test responses and observation notes were documented. Informed consent was obtained from students and guardians, with clear information about voluntary participation and confidentiality protocols.

2.2. Data Analysis and Trustworthiness

Data analysis was conducted in three steps: data reduction, data display, and conclusion drawing (Miles & Huberman, 1994). Responses were coded based on the four indicators of critical thinking. Matrices were developed to compare patterns across students with different ability levels. This process allowed the researcher to identify how each student approached analysis, constructed strategies, made inferences, and evaluated their solutions.

Findings showed consistent variation across student profiles. Common themes included over-reliance on procedures, difficulty with conceptual linking, and limited use of evaluation. To ensure credibility, triangulation was applied by comparing test results, interview transcripts, and observation data. Member checking was conducted by asking participants to verify summaries of their interviews. Peer debriefing with external researchers supported the accuracy and objectivity of the interpretation.

This methodological approach provided a comprehensive lens to understand how students engage in critical thinking within algebraic contexts. The use of validated instruments, ethical safeguards, and multiple data sources ensured that the findings were trustworthy and valuable for informing future instructional practice.

3. Results

This section presents the research findings based on students' performance in solving derivative problems within an algebraic context. The analysis focuses on the four indicators of critical thinking: Analysis, Strategy, Inference, and Evaluation. Data from written responses, interviews, and classroom observations were used to assess students at high, medium, and low levels of critical thinking ability.

3.1 Critical Thinking Performance

The results of this study revealed clear distinctions in student performance based on their level of critical thinking ability. These differences emerged not only in how many of the critical thinking stages each student completed but also in the depth, consistency, and originality of their reasoning within those stages. The four main indicators assessed were Analysis, Strategy, Inference, and Evaluation. Table 3.1 presents a summary of students' overall achievements across these indicators.

Table 3.1 Student Achievement in Critical Thinking Stages

Stage	Indicator Description	Student Observations
Analysis	Ability to interpret mathematical visuals and contextual problems, and explain relevant variables and formulas systematically	High-performing students demonstrated clear understanding of problem structure and could identify key variables.
Strategy	Ability to construct conjectures, develop solution plans, and connect mathematical relationships	Most students at medium and high levels could develop relevant solution strategies using appropriate formulations.
Inference	Ability to infer conclusions from patterns or relationships within mathematical expressions and apply relevant concepts	Only high-level students consistently applied correct reasoning and showed logical inference toward the correct solutions.
Evaluation	Ability to validate the correctness and completeness of solutions by rechecking calculations and logical consistency	Evaluation was the most challenging stage. Only students with high critical thinking skills completed this stage fully.

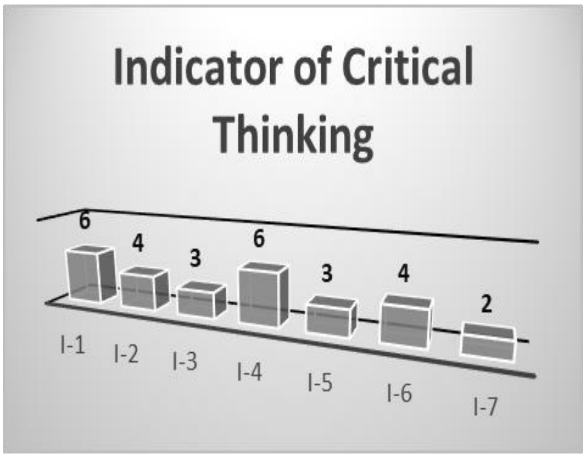

Figure 3.1 Graph of Mathematical Indicator Achievement

As presented in Figure 5, students with high levels of critical thinking were able to engage meaningfully across all four stages. These students displayed not only procedural competence but also the capacity to interpret mathematical contexts accurately, construct coherent strategies, connect concepts across tasks, and critically assess the validity of their solutions. Their approach reflected an understanding of mathematics as a logical and interconnected discipline rather than as a set of isolated procedures. As illustrated in the transcript, one high-level participant explained: *"I always try to make sure the answer fits the real problem. If the width becomes zero, then something must be wrong. I check again until it makes sense"* (Student F, Interview Transcript).

Students at the medium level demonstrated strength in the earlier stages, particularly in analysis and strategy. They were generally able to identify relevant variables and formulate a plausible approach to the problems. However, their performance often declined at the inference and evaluation stages. This decline appeared to stem from limited conceptual fluency and a lack of confidence in making abstract connections. One medium-level participant reflected on this by saying: *"I can find the derivative, but sometimes I do not really understand what the result means. I just do the steps"* (Student A, Interview Transcript). This procedural orientation was further confirmed through classroom observation, where Student A was seen frequently looking toward the teacher for confirmation before proceeding independently (Observation Notes, 27 February 2025).

Students at the low level exhibited the most significant challenges across all four indicators. Their difficulties began with the initial analysis stage, where they often misinterpreted visual prompts or failed to identify essential components of the problem. This lack of clarity in understanding the problem carried over into the strategy stage, resulting in fragmented reasoning and inaccurate attempts to formulate a solution. In many cases, students at this level showed signs of procedural imitation without clear understanding. For instance, Student D stated: *"I just tried the formula we used before, but I am not sure if it fits this one"* (Student D, Interview Transcript).
The classroom observer noted that this student spent a significant amount of time copying from peers and hesitated to continue working after a mistake, which indicates reliance on external cues rather than internal reasoning (Observation Notes, 28 February 2025).

A particularly noteworthy trend across all performance levels was the persistent weakness in the evaluation stage. Even among students who performed relatively well in the earlier stages, few demonstrated the habit of revisiting their work to verify their results or consider the accuracy of their reasoning. Many students accepted their first answer as final, without examining whether it made sense mathematically or contextually. This was evident when a medium-level student remarked: *"No, because I thought once I found x, it must be correct. I do not usually check unless I am told to"* (Student B, Interview Transcript). The observer corroborated this behavior, noting that many students left their answer sheets without attempting to recheck or comment on the feasibility of their solutions, even when prompts were provided by the teacher (Observation Notes, 28 February 2025).

These findings suggest that success in mathematics involves more than following procedures or reaching the correct answer. It requires the ability to engage in thoughtful interpretation, to construct logical pathways toward a solution, to make reasoned inferences, and to assess whether the solution is valid and meaningful. While students may demonstrate competence in isolated stages, the integration of all four indicators is necessary for genuine critical thinking. The performance gaps observed in this study point to the importance of instructional practices that promote conceptual understanding, encourage students to explain their reasoning, and provide structured opportunities to reflect on and evaluate their mathematical processes. These conclusions are triangulated across student responses, teacher observations, and student interviews.

In conclusion, the results reinforce the importance of explicitly teaching and modeling evaluative thinking in mathematics classrooms. Teachers should integrate opportunities for self-assessment, peer review, and justification of reasoning into daily instruction. One mathematics teacher commented during post-lesson reflection: *"Students can differentiate, but the moment you ask why the solution works, many freeze. They are not used to explaining"* (Teacher Interview, 1 March 2025).

By guiding students to view mathematics as a discipline that emphasizes reasoning as much as obtaining the correct results, educators can nurture essential habits of mind that support the development of independent and critical thinkers. The integration of reflective discourse alongside structured justification activities enables students to internalize the complete cycle of mathematical thinking. This includes understanding problems conceptually, constructing logical strategies, drawing inferences, and critically evaluating the validity of their solutions. Such an approach encourages students to engage more deeply with mathematics as a coherent and meaningful process rather than as a set of disconnected procedures.

In conclusion, the results reinforce the importance of explicitly teaching and modeling evaluative thinking in mathematics classrooms. Teachers should integrate opportunities for self-assessment, peer review, and justification of reasoning into daily instruction. By helping students see mathematics as a discipline that values reasoning as much as results, educators can foster the habits of mind necessary for students to become independent and critical thinkers.

3.2. Student Profiles Based on Critical Thinking Levels

To gain a more detailed understanding of students' reasoning patterns, individual performances across the four critical thinking indicators were analyzed. This profile-based analysis highlights not only how many stages each student completed but also how consistently they demonstrated the cognitive skills associated with each stage. Table 6 displays the distribution of indicator achievement among the six selected students, representing high, medium, and low levels of critical thinking ability.

Table 3.2 Indicators of Student Achievement in Critical Thinking

Student	Analysis	Strategy	Inference	Evaluation
F (High)	✓	✓	✓	✓
C (High)	✓	✓	✓	✓
A (Medium)	✓	✓	✗	✗
B (Medium)	✓	✓	✗	✗
D (Low)	✗	✗	✗	✗
E (Low)	✗	✓	✗	✗

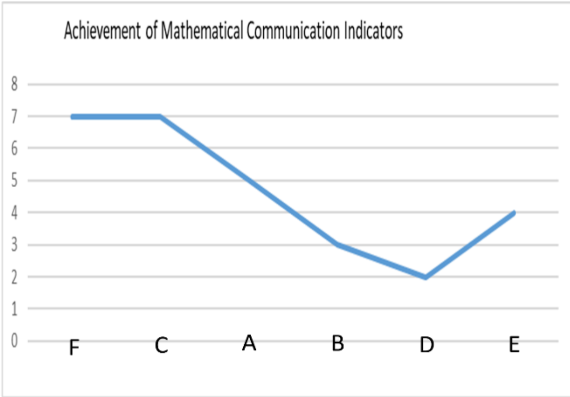

Figure 3.2. Achievement of Critical Thinking Indicators

As shown in Table 6 and illustrated in Figure 6, only students F and C, both from the high-performance group, demonstrated complete mastery across all four critical thinking indicators. These students consistently applied mathematical reasoning from the initial interpretation of the problem to the final evaluation of their solutions. Their approach was organized and reflective, often characterized by clear verbal articulation and adaptive thinking. During the interview, Student F commented: *"I started by checking what information I had, then imagined the box before writing the volume formula. If I got a strange value like zero width, I checked again until it made sense"* (Student F, Interview Transcript). Similarly, Student C remarked: *"I like to draw and label first. Then I test the result to see if it really works. If not, I go back and fix it"* (Student C, Interview Transcript). Classroom observations confirmed that these students independently checked their answers and provided verbal explanations without being prompted (Observation Notes, 1 March 2025). Their thinking revealed an internalization of mathematical principles that extended beyond surface-level manipulation.

Students A and B, categorized at the medium level, showed promising performance in the first two stages: analysis and strategy. They were able to identify relevant variables and construct solution plans based on standard procedures. However, they encountered significant obstacles when engaging with inference and evaluation. Their work demonstrated sound initial structure, but this structure often collapsed under the demands of deeper reasoning. Student A reflected: *"I can build the function and get the derivative, but I am not sure what to do after that"* (Student A, Interview Transcript). Student B echoed this sentiment: *"I understand the method, but I do not always know how to explain why it works"* (Student B, Interview Transcript). Observation notes revealed that both students hesitated during tasks that required independent justification, frequently looking around for cues from peers or waiting for the teacher to confirm their next step (Observation Notes, 27 February 2025). Although they possessed procedural fluency, their reasoning often lacked confidence and conceptual clarity, particularly when navigating unexpected problem requirements.

Students D and E, representing the low-performing group, demonstrated minimal success across all indicators. Student D was unable to engage meaningfully with any of the stages and struggled to identify basic information in the problem. Their written work lacked coherence and often deviated from the intended mathematical structure. When asked how they approached the problem, Student D responded: *"I do not really know. I just wrote what I remembered from before"* (Student D, Interview Transcript). This statement reflected the absence of a strategic or reflective approach. Observational notes further recorded that Student D gave up midway during the task and began copying from others without attempting to revise or explain (Observation Notes, 28 February 2025).

Student E showed slight success in the strategy stage, indicating partial recall of procedural steps, but lacked integration of those steps into a meaningful plan. When prompted to explain their method, Student E replied: *"I think this is what we did last time, but I'm not sure if it fits here"* (Student E, Interview Transcript). Observation records described Student E as disengaged during peer discussions and reliant on trial-and-error without checking the logical consistency of the result (Observation Notes, 28 February 2025). These behaviors underscored gaps not only in mathematical comprehension but also in metacognitive regulation and task persistence.

Across all profiles, a consistent pattern emerged: while some students could perform isolated procedures, they often lacked the conceptual depth to understand why those procedures worked or when they should be applied. This distinction became increasingly clear in the inference and evaluation stages. Students at higher levels demonstrated awareness of the logical flow between stages and the importance of validating their results. In contrast, students at the medium and low levels tended to follow patterns without full comprehension, which compromised the integrity of their conclusions.

These profiles confirm that critical thinking in mathematics is not a fixed trait but a dynamic interaction of understanding, reasoning, and reflection. Students who reached the highest level of performance were those who internalized the logic behind the tasks and could verbalize or justify their decisions coherently. As Student C explained: *"It is not just about the right answer, but why it works the way it does. I try to imagine the shape and test the logic"* (Student C, Interview Transcript).

The data from individual profiles reinforce the broader conclusion that mastery of critical thinking in mathematics involves more than getting correct answers. It involves the ability to understand problems deeply, devise strategies purposefully, infer meaning with clarity, and evaluate solutions with confidence and logic. These distinctions emphasize the need for instructional models that integrate all four indicators in a balanced and intentional way, supporting students as they move from surface-level processing to deeper mathematical reasoning. As noted by the classroom teacher during reflective discussion: *"Students need to learn how to ask themselves 'Does this make sense?' That habit takes time and training"* (Teacher Interview, 1 March 2025).

3.3 In-Depth Findings Across Indicators

3.3.1 Analysis and Strategy

In the analysis stage, students were expected to interpret both visual and contextual information, identify variables, and translate the task into a structured mathematical model. High-performing students showed the strongest ability to fulfill this expectation. They were able to dissect the problem, extract essential components such as dimensions, constraints, and relationships between variables, and convert them into algebraic expressions. This initial interpretation served as a foundation for accurate and strategic problem solving.

An example task involved designing an open box by cutting equal squares from each corner of a rectangular cardboard, then determining the maximum volume of the resulting shape. Students needed to translate this scenario into an algebraic volume function by identifying the expressions for length, width, and height.

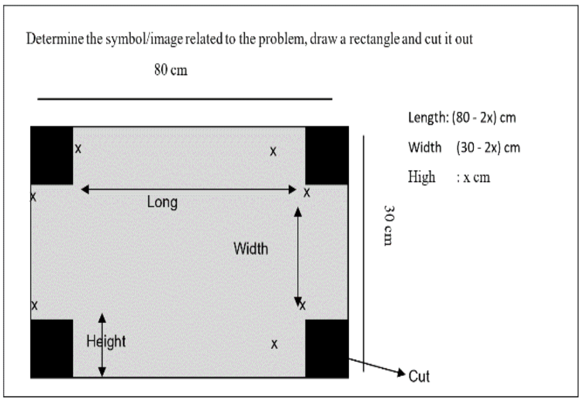

Figure 3.3 Analysis Stage Indicators

High-level students navigated this task with clarity. Their representations were precise and coherent, revealing a solid grasp of algebraic structure. They demonstrated control over the translation of geometric dimensions into symbolic expressions, and this accuracy enabled them to proceed confidently to the strategy stage. In contrast, most medium-level students reached this stage but required external prompts to articulate relationships and to avoid confusion in expressing side reductions. Their representations were often incomplete or mechanically formed, indicating partial understanding rather than conceptual certainty. Low-level students had the most difficulty, frequently overlooking important constraints or misapplying the visual cues, which led to flawed or meaningless algebraic models.

In the strategy stage, students were expected to differentiate the volume function, identify critical points, and interpret their meaning in the context of the problem. High- and medium-level students generally succeeded in performing the necessary differentiation. However, only students at the higher level demonstrated confidence in interpreting the significance of their results and identifying constraints that affected their solutions. They were able to determine which values of x were valid within the context of the problem and to justify the rejection of mathematically correct but contextually invalid outcomes.

Creating Math relationships

Suppose the volume of a beam V(x),

Then V(x) = Length x Width x Height

= (80 - 2x) (30 - 2x) x

= (2400 - 160x - 60x + 4x2) x

= (4x2 - 220x + 2400) x

= 4x3 - 220x2 + 2400x

So the volume formula in variable x is V(x) = 4x3 - 220x2 + 2400x

Figure 3.4 Strategy Stage Indicators

This stage revealed a crucial distinction between procedural knowledge and strategic thinking. While many students could perform symbolic manipulation to find derivatives, few could explain what those results meant or apply them logically within problem constraints. Students at the medium level, for example, often failed to recognize that a length or width of zero would invalidate the box. This inability to connect formal results with practical meaning reflected a lack of conceptual grounding. Among low-level students, strategy construction was inconsistent or entirely absent. They tended to attempt memorized procedures or skip this step altogether when initial interpretations were unclear.

3.3.2. Inference and Evaluation

Inference emerged as the most cognitively demanding stage. It required students to generalize from prior concepts, identify patterns in their algebraic manipulation, and logically determine which results could be interpreted as solutions to the posed problem. High-level students displayed consistency in this stage. They drew on previously acquired knowledge, connected it to the derivative functions they had constructed, and justified their conclusions based on mathematical constraints.

Length x for maximum volume

For maximum volume, V'(x) = 0

V(x) $= 4x^3 - 220x^2 + 2400x$

V'(x) $= 12x^2 - 440x + 2400$

$0 = 12x^2 - 440x + 2400$

$0 = 4(3x^2 - 110x + 600)$

$0 = 4(3x - 20)(x - 30)$

From the above factors obtained

3x - 20 = 0	x - 30 = 0
3x = 20	x = 30
x = 20/3	

Figure 3.5 Inference Stage Indicator

Students at the medium level often began this process correctly but struggled to reach accurate conclusions due to misapplication of algebraic rules or misunderstanding of quadratic characteristics. They frequently attempted to use the correct formulas but failed to contextualize or justify the outcome. This difficulty was compounded by computational errors or overlooked domain limitations. Students in the low category rarely demonstrated meaningful inference. They either copied from others, made speculative guesses, or offered numerical answers without establishing how those values were derived from their previous work.

The evaluation stage, across all profiles, was the most underdeveloped. High-level students were the only participants who consistently returned to their previous steps to verify the correctness of the results. They checked whether their values of x made sense geometrically, confirmed that lengths and widths remained positive, and recalculated volumes when needed. Importantly, they demonstrated the ability to reject mathematically valid results that were contextually invalid, such as a box dimension becoming zero.

The value x = 30 does not satisfy because the width of the rectangle is 30,

And if you cut a corner 30 cm long, it won't be enough.

Then the value of x that satisfies is x = 20/3.

Max Volume

$= 4x^3 - 220x^2 + 2400x$

= 4 (20/3)^(3)-220(20/3)^(2)+2400(20/3)

= 4(8000/27)-220(400/9)+2400(20/3)

= 32000/27-264000/9+432000/27

= (32000-264000+432000)/27

= 200000/27

= 7407 cm^3

Figure 3.6 Evaluation Stage Indicator

In contrast, medium-level students displayed minimal awareness of the need to re-evaluate their answers. Even when inconsistencies were apparent, they accepted results at face value and showed no indication of verifying accuracy. Students at the low level either left this stage unattempted or provided vague and unsupported conclusions. When prompted, many were unable to explain why a result might be logically incorrect. This reinforces the interpretation that evaluation, though critical to problem solving, is often overlooked in students' mental routines. Many seem to equate solving a problem with completing a series of steps, rather than reflecting on whether those steps lead to a reasonable and justified conclusion.

These findings highlight the need for greater instructional focus on the evaluation stage. The ability to critique reasoning, identify errors, and justify decisions does not develop automatically through procedural practice. It requires deliberate opportunities for explanation, reflection, and metacognitive engagement. The study showed that even students who could differentiate and interpret results often failed to assess their reasoning critically. This gap between doing and thinking reveals a key limitation in mathematical performance at the secondary level.

In summary, the analysis of these indicators highlights that critical thinking in mathematics is an integrated process. It involves more than isolated skills; it reflects a student's capacity to move fluidly from interpretation to justification and from computation to reflection. Among all four indicators, evaluation remains the most essential yet least practiced. Strengthening this component could significantly improve students' overall mathematical reasoning and problem-solving capacity. Instructional strategies that promote explanation, peer discussion, and iterative review may provide the necessary structure to help students internalize the full cycle of mathematical thinking.

5. Discussion

This study identified significant differences in students' performance across the four indicators of critical thinking, namely Analysis, Strategy, Inference, and Evaluation, when solving algebraic problems involving derivatives. High-achieving students such as F and C consistently demonstrated competence across all four indicators. Medium-level students showed reasonable ability in Analysis and Strategy but struggled with Inference and Evaluation. Low-level students encountered difficulties in all stages, with particular weaknesses in problem interpretation and the logical validation of their responses. These findings align with prior studies emphasizing the importance of conceptual understanding and reflective reasoning in mathematics learning (Sumarmo, 2006; Jumaisyaroh & Hasratuddin, 2016; Ahdhianto et al., 2020; Kaya & Aydin, 2016).

The results reinforce the view that critical thinking in mathematics is not limited to computational procedures but involves interpreting contexts, constructing logical strategies, and critically examining conclusions (Wilkinson et al., 2018; Zakiri et al., 2018). High-performing students exhibited metacognitive awareness by verifying the contextual validity of their answers, reflecting on the coherence of their reasoning, and revisiting assumptions. For example, Student F stated the need to ensure that answers made logical sense, which reflects internalized habits of evaluation.

In contrast, medium-level students often reached the correct derivative but lacked clarity in interpreting results or explaining their significance. Student A, for instance, admitted performing steps without fully understanding their meaning. This procedural reliance suggests limited conceptual fluency, a pattern observed in other studies as well (Kurniawan et al., 2017; Aprilianingrum & Wardani, 2021). Low-level students, such as D and E, demonstrated minimal ability to engage with the tasks meaningfully. They tended to imitate procedures without understanding, sought confirmation from peers, and struggled with mathematical language and representation. These observations are consistent with findings from Shannon and Austin (1992), who highlighted the obstacles students face when solving unfamiliar problems without conceptual support.

The most underdeveloped dimension across all profiles was the evaluation stage. Students often accepted their first answer as correct without reviewing the logic or feasibility of their solutions. Even those who succeeded in the earlier stages rarely reflected on the validity of their final answers unless prompted. This lack of evaluative reasoning confirms that procedural competence alone does not equate to critical thinking and that students require structured opportunities to develop reflection and justification skills (Pourdavood et al., 2020; Kusumah et al., 2020).

This study addresses a critical gap in the literature by centering the evaluation indicator within the context of algebra problem-solving in Indonesia. Prior research has frequently examined general critical thinking frameworks but has rarely investigated how students validate mathematical reasoning, particularly in derivative-based tasks. This scarcity highlights the significance of this study's focus on cognitive and metacognitive processes involved in the evaluation stage of problem-solving. Evidence indicates that high-achieving students can distinguish between mathematically valid and contextually invalid answers, enriching our understanding of how critical thinking operates within secondary mathematics classrooms (Shodikin et al., 2022; , Safitri & Suryani, 2022).

Metacognition, defined as the awareness and regulation of one's thought processes, plays a pivotal role in mathematical problem-solving, particularly in an educational setting. Students who engage in metacognitive strategies are better equipped to manage their learning processes, enhancing their problem-solving abilities (Rambe & Asmin, 2019; , Murtiyasa & Anggraini, 2022). For instance, research has demonstrated that students with higher metacognitive awareness exhibit superior mathematical problem-solving skills, suggesting that metacognitive strategies should be integrated into instructional practices to improve student outcomes (Santoso et al., 2019; , Kathayat, 2024). High-achieving students not only show elevated levels of metacognitive engagement but also display strategic behaviors that enable them to evaluate their work critically, thereby promoting deeper understanding and cognitive flexibility (Ramlah et al., 2024; Restini et al., 2023).

The novelty of this study is underscored by its examination of metacognitive activities across different levels of mathematical ability. It aligns with previous findings indicating that students' metacognitive skills emerge at each stage of problem-solving, as suggested by models like Polya's problem-solving stages (Restini et al., 2023). The ability to engage in reflective practices—evaluating one's thought processes and problem-solving strategies—provides these students a tangible advantage, enabling them to navigate complex tasks more effectively (Pennequin et al., 2010; , Rosikhoh et al., 2022). Furthermore, the evidence that high-achieving students can decipher context-rich problems affirms the importance of cultivating metacognitive skills to foster both critical thinking and mathematical reasoning capabilities (Darmawan et al., 2019; , Mas'ud et al., 2018).

The findings carry important implications for both theory and instructional practice. Theoretically, this study confirms that critical thinking is a dynamic and integrated process involving comprehension, strategy formulation, inference, and self-assessment. Pedagogically, it suggests that mathematics instruction should go beyond procedural teaching and instead nurture students' reasoning and reflection. Teachers must create spaces where students are encouraged to explain their thought processes, justify their conclusions, and question the logic of their outcomes (Chasanah et al., 2020; Elliott et al., 2001). Training educators to facilitate evaluative discourse and integrate self-assessment into lessons may help bridge the gap between knowing and understanding.

The importance of nurturing critical thinking through meaningful evaluation also aligns with national education reforms in Indonesia that emphasize higher-order thinking. Equipping students with skills to critique and validate their reasoning is essential for preparing them to face complex real-world problems and to compete at an international level (NTCM, 2000; Liu, 2020).

Future research should investigate how students' evaluation abilities develop over time and whether specific instructional strategies can enhance this dimension of critical thinking. Longitudinal studies could track the progression of evaluative reasoning from junior to senior high school. Further investigation could also examine the effectiveness of digital tools or collaborative learning models in promoting reflection and reasoning. Studies might expand to other mathematical domains such as geometry or statistics to explore whether similar patterns of critical thinking emerge. In addition, culturally responsive pedagogical approaches and their influence on students' evaluative thinking in mathematics warrant further exploration, particularly in diverse educational contexts across Indonesia (Kurniawan et al., 2024; Richardo et al., 2023).

4. Conclusion

This study highlights that students' performance in algebra problem solving varies significantly across the four critical thinking indicators, with evaluation emerging as the most underdeveloped dimension. High-achieving students consistently demonstrated a robust integration of analysis, strategic formulation, logical inference, and evaluative reasoning, showing not only procedural fluency but also the capacity to reflect, justify, and revise their mathematical solutions. In contrast, medium-level students often relied on procedural routines without conceptual depth, particularly struggling to explain or assess their answers meaningfully, while low-performing students showed fragmented reasoning and a heavy dependence on imitation.

A key finding is that even students who can perform derivations or identify correct solutions frequently lack the metacognitive awareness to validate those results within real-world contexts. This gap between execution and reflection reveals a pressing need to reposition evaluation as central to mathematical learning. The novelty of this research lies in its focused exploration of how Indonesian students engage with the evaluation stage, an area rarely isolated in previous studies, particularly within algebraic contexts involving derivative problems.

The study's implications extend to instructional design, urging educators to foster classroom cultures that value reasoning as much as correctness and to equip learners with reflective tools that support deeper mathematical understanding. To advance this agenda, future research should explore how evaluative thinking evolves through sustained intervention, how collaborative and digital environments can enhance metacognitive engagement, and how culturally responsive strategies may empower diverse learners to navigate mathematical complexity with confidence and critical insight.

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