

From Play to Mathematical Discourse: Transforming Multiplication Learning Through GASING Card Games and Deep Learning

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ABSTRACT

Mathematics becomes meaningful when children can explain, represent, and apply numerical relationships rather than merely reproduce procedures. Yet elementary instruction often remains teacher-centred, while previous GASING and game-based studies have commonly relied on small samples, single-school settings, digital media, or single pretest–posttest comparisons that cannot distinguish sustained change from ordinary fluctuation. This study examined an integrated model of pedagogical deep learning operationalised through GASING (*Gampang, Asyik, dan Menyenangkan*; Easy, Enjoyable, and Fun) and a physical mathematics card game to strengthen fourth-grade students' multiplication problem-solving ability. A quantitative quasi-experimental interrupted time-series design involved 132 students from six elementary schools in Banyuwangi Regency. Three pre-intervention and three post-intervention measurements were administered using a context-based problem-solving test, supported by implementation-fidelity observations. Data were analysed using descriptive statistics, segmented regression, paired and non-parametric comparisons, normalized gain, and Cohen's *d*. Results revealed a stable baseline ($M = 21.34\text{--}23.95$) followed by an immediate and sustained increase after the intervention ($M = 74.54\text{--}81.52$). Segmented regression identified a large level change ($\beta_2 = 46.83$, $p < .001$), while the overall normalized gain was high ($g = 0.71$), the effect size was very large ($d = 2.93$), and implementation fidelity reached 90.5%. Improvements occurred across all six schools. These findings show that structured physical play can deepen conceptual learning, mathematical discourse, collaboration, and reflection. The study offers an accessible, low-cost, and methodologically robust model for improving numeracy and advancing learner-centred mathematics reform within and beyond the Merdeka Curriculum.

1. Introduction

Mathematics becomes meaningful when children can interpret, represent, explain, and apply numerical relationships, not merely produce correct answers. Numeracy therefore extends beyond computation to understanding information, making decisions, and solving authentic problems (Díaz & Butter, 2021; Vizcaya & Velásquez, 2024). This view aligns with *Alfabetización Matemática*, which frames mathematics as meta-learning and empowerment (Coronata & Alsina, 2021; Hernández, 2022). Elementary mathematics should consequently develop problem-solving, reasoning, proof, communication, connection, and representation (Beltrán-Pellicer & Alsina, 2022; Vásquez et al., 2021). Contextual tasks and modelling further strengthen numeracy by connecting abstract ideas with learners' experiences (Coronata & Alsina, 2021; Díaz & Butter, 2021).

Numbers remain silent until learners can explain the relationships they represent. Numeracy is therefore mathematical discourse constructed through language, symbols, representations, gestures, and interaction (Planas, 2018). Problem-solving develops this discourse because learners must formulate, employ, interpret, and communicate mathematics in authentic contexts (OECD, 2023; Polya, 2004), while strengthening critical thinking, creativity, communication, collaboration, citizenship, and character (Fullan et al., 2018). Yet Indonesian students still struggle with contextual interpretation, reasoning, and knowledge transfer (OECD, 2023). Teacher-centred instruction may produce correct calculations without lasting understanding (Hattie, 2023), while persistent difficulties can increase mathematics anxiety (Barroso et al., 2021). Indonesia therefore promotes deep learning through reflection, critical thinking, meaningful application, and sustained competence.

Previous research offers valuable foundations, but its evidence remains fragmented. Deep-learning frameworks encourage learners to connect ideas, investigate relationships, reflect, and transfer knowledge beyond routine procedures (Fullan et al., 2018; Hattie, 2023). Problem solving followed by instruction can strengthen understanding when teachers build on attempted solutions, errors, and comparisons (Loibl et al., 2017). Concrete materials and visual representations also support movement from physical experience to symbols when their connections are explicitly mediated (Fyfe & Nathan, 2019; Kim, 2020). Concrete-representational-abstract instruction can similarly improve understanding, problem-solving, and transfer, depending on content, sequencing, and teacher guidance (Kaya & Yildiz, 2024; Kokkonen & Schalk, 2021; Prosser & Bismarck, 2023). However, these elements are usually examined separately rather than integrated into a culturally relevant Indonesian elementary mathematics model.

Game-based learning can improve mathematics achievement, motivation, engagement, problem-solving, and higher-order thinking when aligned with curricula and supported by feedback, scaffolding, and teacher facilitation (Byun & Joung, 2018; Chen et al., 2022; Hui & Mahmud, 2023; Hussein et al., 2022; Sailer & Homner, 2020; Tokac et al., 2019; Vankúš, 2021; Wang et al., 2022; Yu et al., 2021). Primary studies also report gains in number sense, strategic flexibility, geometry, early algebra, and mathematical interest when games connect with formal concepts (Brezovszky et al., 2019; Hulse et al., 2019; Nand et al., 2019; Sun et al., 2021; Yeh et al., 2019). Online games support interaction and feedback, although their quality varies (Hidayat et al., 2024). In Indonesia, *GASING*, *Gampang*, *Asyik*, dan *Menyenangkan*, combines progression, concrete experience, pattern recognition, practice, and enjoyment, with positive outcomes for achievement and problem-solving (Hayati et al., 2024; Rumajar et al., 2024). Nevertheless, many studies involve small samples, single schools, general achievement measures, or one-group pretest-posttest designs. Digital games also dominate, leaving physical card games and face-to-face mathematical discourse underexplored. No established multisite study has integrated pedagogical deep learning, *GASING*, collaborative card play, and Polya-oriented problem-solving, while single pretest-posttest comparisons cannot adequately distinguish sustained change from ordinary fluctuation (Bernal et al., 2017).

Responding to these gaps, this study operationalises pedagogical deep learning through *GASING* and a physical mathematics card game. Its niche lies in positioning play as a semiotic and social space for exploration, pattern discovery, strategic discussion, explanation, reflection, and symbolic generalisation. Students move among card manipulation, spoken explanations, visual patterns, written strategies, and formal symbols while

investigating multiplication. Its novelty lies in integrating deep learning, *GASING*, collaborative play, mathematical discourse, and Polya-oriented problem-solving within one instructional architecture. Methodologically, the study uses three pre-intervention and three post-intervention measurements with 132 fourth-grade students from six elementary schools in Banyuwangi Regency, enabling analysis of baseline stability, immediate change, subsequent development, and consistency across observations (Bernal et al., 2017; Hayati et al., 2024; Rumajar et al., 2024).

The study is theoretically, pedagogically, methodologically, and curricularly significant. It connects deep learning, representational progression, mathematical discourse, game-based learning, and problem-solving; offers teachers a low-cost alternative to formula-centred instruction; and relates repeated outcome measurements to implementation fidelity. It also supports conceptual understanding, numeracy literacy, learner agency, critical reasoning, collaboration, meaningful learning, and reflection. Accordingly, the study aims to analyse how pedagogical deep learning is implemented through a *GASING*-based mathematics card game and identify changes in fourth-grade students' mathematical problem-solving ability following the intervention.

Two research questions guide the investigation. RQ1: How is pedagogical deep learning implemented through a *GASING*-based mathematics card game in fourth-grade multiplication instruction? This question examines participation, conceptual exploration, pattern discovery, mathematical communication, collaboration, reflection, teacher scaffolding, and game engagement. RQ2: What changes occur in students' mathematical problem-solving ability following the intervention, as indicated by levels and trends across repeated pre-intervention and post-intervention measurements? This question assesses students' abilities to understand problems, formulate strategies, execute solutions, and evaluate results according to Polya's problem-solving stages (Polya, 2004). Together, the questions connect implementation quality with performance changes over time.

The study shows that play can remain intellectually demanding when structured through representation, dialogue, strategic reasoning, reflection, and conceptual generalisation. It offers an evidence-informed model for teaching multiplication through connected numerical relationships instead of memorised facts. Low-cost physical materials can promote discussion and contextualised numeracy without advanced digital infrastructure, while learner-centred curricula can integrate conceptual depth, enjoyment, collaboration, and reflection. Repeated measurements strengthen evaluations of classroom innovation. Ultimately, play becomes problem-solving through discourse as children construct, communicate, test, evaluate, and refine mathematical ideas.

2. Literature Review

2.1 Pedagogical Deep Learning and Mathematical Problem Solving

Pedagogical deep learning shifts mathematics instruction from memorizing rules and imitating procedures toward constructing, connecting, and applying mathematical knowledge. Students relate new concepts to prior understanding, explore relationships, reflect on their reasoning, and transfer knowledge to unfamiliar situations. Fullan et al. (2018) link this orientation to six competencies: character, citizenship, collaboration, communication, creativity, and critical thinking. In Indonesia, these competencies are reflected in meaningful, mindful, and joyful learning, which supports conceptual depth, active participation, reflection, and positive learning experiences (Feriyanto & Anjariyah, 2024).

This perspective aligns with evidence that explicit instruction becomes more effective when combined with active engagement, feedback, reflection, and knowledge application (Hattie, 2023). Mathematics is therefore viewed as an interconnected system of concepts, representations, patterns, relationships, and reasoning processes rather than a collection of procedures.

Pedagogical deep learning is grounded in constructivist perspectives. Piaget and Inhelder (1969) explain knowledge development through assimilation and accommodation, while Vygotsky (1978) emphasizes language, social interaction, cultural tools, and scaffolding. These principles are especially relevant to fourth-grade learners, who benefit from concrete materials, visualization, guided exploration, and interpersonal explanation. However, manipulatives promote understanding only when learners connect physical actions with visual, verbal, and symbolic representations. Concreteness fading therefore supports a mediated progression from concrete experiences to abstract forms (Fyfe & Nathan, 2019; Kim, 2023). Its effectiveness depends on explicit representational connections, suitable mathematical content, and teacher guidance rather than the presence of materials alone (Kaya & Yildiz, 2024; Kokkonen & Schalk, 2021; Prosser & Bismarck, 2023). Movement across concrete, representational, and abstract forms can integrate procedural fluency with conceptual understanding, although the sequence should remain responsive to learners and mathematical topics (Fyfe & Nathan, 2019; Kokkonen & Schalk, 2021).

Mathematical problem solving demonstrates deeper understanding by integrating reasoning with cognitive, motivational, and affective processes across contexts (Wang & Guo, 2026). Motivation and self-regulated learning predict performance, while mathematical self-efficacy and thinking operate as proximal predictors and mediators (Wang & Guo, 2026). Affective factors and contextual or multimodal

representations further influence learning outcomes. Technology-enhanced open tasks, immediate feedback, and real-world objects can strengthen logical competencies (Weinhandl et al., 2020). Cross-disciplinary approaches such as DECA, involving discovery, extraction, creation, and assembly, may also support computational thinking, although a study of 59 learners found mixed evidence of transfer from programming (Ezeamuzie, 2023).

Retrieval practice similarly improves near and content transfer in mathematics (Schwerter et al., 2026). Polya's (2004) framework organizes problem solving into understanding the problem, devising and implementing a plan, and reviewing the solution. These processes require comprehension, representation, strategy selection, calculation, explanation, and evaluation. They also support mathematical literacy, which involves formulating, employing, and interpreting mathematics in real-world situations (OECD, 2023). Effective elementary instruction should therefore integrate conceptual understanding, procedural fluency, strategic flexibility, communication, reflection, and knowledge transfer.

2.2 GASING and Game-Based Learning as an Integrated Pedagogy

The GASING method provides an instructional structure consistent with pedagogical deep learning. GASING, an acronym for *Gampang, Asyik, dan Menyenangkan* or Easy, Enjoyable, and Fun, was developed by Yohanes Surya to make mathematics accessible through gradual, meaningful, and engaging experiences. It progresses from simple to complex concepts and from concrete experiences to abstract notation while emphasizing numerical patterns, mental calculation, tiered practice, and repeated success. Instead of presenting formulas as fixed procedures, GASING encourages learners to identify relationships and construct efficient strategies, thereby strengthening numerical intuition and reducing reliance on memorized algorithms.

Studies indicate that GASING can improve elementary students' mathematical problem-solving skills and produce stronger outcomes than direct or lecture-based instruction in integer multiplication (Hayati et al., 2024; Rumajar et al., 2024). However, existing studies commonly use small samples, limited topics, single-school settings, short interventions, and conventional pretest-posttest designs. They also focus primarily on achievement and computation rather than conceptual exploration, strategic communication, reflection, and comprehensive problem-solving processes.

Game-based learning can strengthen GASING by engaging students in purposeful challenges involving problem solving, strategy testing, decision making, and communication, particularly through paired activities and real-world representations (Fernandes et al., 2023;

Wu, 2024). Its effectiveness depends on aligning game rules, goals, feedback, representations, and interactions with intended learning outcomes. This principle is supported by feasible gamified problem-based designs with a mean evaluation of 3.4 out of 4 (Panis et al., 2020) and augmented-reality games that promote meaningful learning through accessible representations (Fernandes et al., 2023).

Studies have also reported positive usability and intention to use gamified tutoring systems, although mobile adaptation and further design refinement remain necessary (Pereira et al., 2023). An augmented-reality air-quality game involving 27 children similarly produced learning gains and positive acceptance (Fernandes et al., 2023). More broadly, game-based learning is associated with achievement, motivation, engagement, problem solving, critical thinking, confidence, persistence, cooperation, and mathematical communication, although outcomes vary according to design quality, instructional duration, teacher support, learner characteristics, and curricular alignment (Chen et al., 2022; Sailer & Homner, 2020; Wang et al., 2022; Yu et al., 2021). Mathematics-focused studies further show that games can outperform conventional instruction, develop adaptive number knowledge and strategic flexibility, and support cognitive and affective growth (Brezovszky et al., 2019; Hui & Mahmud, 2023; Tokac et al., 2019). Nevertheless, enjoyment alone is insufficient; gameplay must require mathematical reasoning and connect students' actions with formal concepts.

Teacher scaffolding is therefore essential for helping learners interpret tasks, compare strategies, explain patterns, identify misconceptions, and generalize mathematical relationships while retaining responsibility for their thinking. Whole-class and individualized scaffolding can improve exploration, interest, learning, and arithmetic performance in primary mathematics games (Sun et al., 2021), consistent with Vygotsky's (1978) view of socially mediated learning. Physical number cards are particularly suitable for fourth-grade learners because they make multiplication structures visible through repeated groups, number pairs, combinations, and alternative strategies.

Collaborative card activities also create a low-risk environment for testing ideas, correcting mistakes, revising strategies, and communicating mathematical reasoning. Such conditions are important because mathematics anxiety is associated with lower achievement and reduced willingness to undertake challenging tasks (Barroso et al., 2021). However, research remains dominated by digital games, gamification platforms, and technology-rich environments (Chen et al., 2022; Hidayat et al., 2024; Hussein et al., 2022). Low-cost physical games that promote face-to-face explanation, negotiation, gesture, and collaborative manipulation therefore remain comparatively underexplored.

2.3 Research Gaps, Novelty, and Theoretical Implications

Several gaps remain in the literature. Pedagogical deep learning is rarely integrated into a clearly sequenced elementary mathematics intervention, while GASING studies mainly emphasize arithmetic achievement rather than comprehensive problem solving. Game-based research also focuses largely on digital tools, leaving low-cost physical games and mathematical discourse underexplored. In addition, representation, scaffolding, collaboration, communication, gameplay, and reflection are often studied separately, and single pretest-posttest designs provide limited evidence of stable instructional change.

This study addresses these gaps by integrating pedagogical deep learning, GASING, a physical mathematics card game, and Polya-oriented problem solving. The model combines card manipulation, pattern exploration, collaborative strategy development, explanation, reflection, and symbolic generalization. Using multiplication, students connect concrete objects, visual patterns, spoken reasoning, written strategies, and formal symbols. An interrupted time-series design involving 132 fourth-grade students from six schools, with three pre-intervention and three post-intervention measurements, enables analysis of baseline stability and performance development. The model connects constructivism, representational progression, game-based learning, scaffolding, and mathematical problem solving. It offers a low-cost approach that supports conceptual understanding, numeracy, communication, collaboration, critical thinking, strategic flexibility, and learner agency.

H₀: There is no change in the pattern of students' mathematical problem-solving ability following the implementation of pedagogical deep learning through the GASING method based on a mathematics card game.

H₁: There is a change in the pattern of students' mathematical problem-solving ability following the implementation of pedagogical deep learning through the GASING method based on a mathematics card game.

3. Method

3.1 Research Design

This study adopted a quantitative approach with a quasi-experimental design. The quantitative approach was selected because the study sought to measure change in students' mathematical problem-solving ability following the intervention using numerical data analysed statistically. A quasi-experimental design was employed because the researchers did not randomize participants at the individual level; Instead, intact classes were used, a common approach in educational research that preserves the natural conditions of school-based learning.

The specific design was an Interrupted Time Series (ITS) design, one of the strongest quasi-experimental designs available when a control group cannot be constituted, because it allows the researcher to identify both a change in level and a change in trend following an intervention. The design comprised six measurement occasions: three before the intervention

(O₁, O₂, O₃) and three after the intervention (O₄, O₅, O₆), with the instructional intervention (X) interposed between O₃ and O₄. Relative to the ordinary pretest-posttest design, the ITS design is able to detect level change, trend change, and the stability of the intervention effect, thereby permitting a more comprehensive analysis of instructional impact.

Table 1. Interrupted Time Series Design

Phase	Symbol	Measurement / Treatment
Pre-intervention	O ₁ O ₂ O ₃	Pretest 1, 2, 3
Intervention	X	Deep learning via GASING + card game
Post-intervention	O ₄ O ₅ O ₆	Posttest 1, 2, 3

3.2 Setting, Population, and Sample

The study was conducted during the odd semester of the 2026/2027 academic year in six elementary schools in Banyuwangi Regency that had implemented the Merdeka Curriculum. The population comprised all fourth-grade students who were targets of Merdeka Curriculum mathematics instruction in the regency. The sample was selected through purposive sampling according to the following criteria: the school

implemented the Merdeka Curriculum; it had teachers who had completed numeracy training; it was willing to host the research; and it supported the use of innovative instructional media. A sample exceeding 100 participants is adequate for regression and time-series analysis in educational research. The total sample comprised 132 fourth-grade students; Table 2 reports the distribution across schools.

Table 2. Distribution of the Research Sample

No.	School	n	%
1	SDN 1 Kalibaru Kulon	32	24.2
2	SDN 2 Pakis	25	18.9
3	SDN 2 Alasrejo	25	18.9
4	SDN Bakungan	20	15.2
5	SDN 4 Ketapang	19	14.4
6	SDN 5 Kaligung	11	8.3
	Total	132	100.0

3.3 Variables and Operational Definitions

The study involved two principal variables. The independent variable (X) was deep learning implemented through the GASING method based on a mathematics card game a mathematics learning process that integrates conceptual exploration, reflective activity, the use of number patterns, the concrete–semi-concrete–abstract sequence, and a card-game activity. The dependent variable (Y) was students' mathematical

problem-solving ability, operationalized as the capacity to solve mathematical problems through the four Polya phases. Implementation fidelity was characterized by six indicators (active student involvement, conceptual exploration, the discovery of number patterns, reflective activity, collaboration, and learning motivation), while problem-solving ability was characterized by four indicators (understanding the problem, planning a strategy, executing the strategy, and evaluating the result).

3.4 Instructional Procedure

The instructional intervention was delivered in six stages. In Stage 1 (Orientation), the teacher explained the learning objectives and the rules of the mathematics card game. In Stage 2 (Concrete Exploration), students used number cards to form number pairs. In Stage 3 (Pattern Discovery), students identified the multiplication patterns arising from the cards. In Stage 4 (Group Discussion), students discussed the strategies they had used. In Stage 5 (Reflection), students articulated their reasoning processes. In Stage 6 (Generalization), the teacher helped students consolidate the mathematical concept under study. Throughout, the teacher functioned as a facilitator, providing scaffolding when students encountered difficulty, in accordance with the principles of deep learning.

3.5 Data Collection and Instruments

Data were collected through a mathematical problem-solving test administered on six occasions (three pretests and three posttests). The test comprised ten context-based constructed-response items mapped to the four Polya indicators, with a maximum score of 100. Item content validity was established by three experts, a mathematics-education specialist, an educational-evaluation specialist, and a practising elementary mathematics teacher, and quantified using Aiken's V , with $V \geq 0.80$ interpreted as highly valid. Test reliability was estimated using Cronbach's alpha, with $\alpha \geq 0.80$ interpreted as high. In addition, structured observation was used to assess implementation fidelity across teacher activity, student activity, use of the game, group interaction, and reflection, rated on a four-point scale; documentation (photographs, teaching materials, samples of student work, and observation sheets) was also collected.

Table 3. Test Blueprint for the Mathematical Problem-Solving Instrument

Indicator	Sub-indicator	Item
Understanding the problem	Identifying information given	1, 2
Understanding the problem	Identifying information sought	3
Planning a strategy	Determining the operation	4, 5
Executing the strategy	Completing the computation	6, 7
Evaluating the result	Re-checking the answer	8
Evaluating the result	Drawing a conclusion	9, 10

3.6 Data Analysis

Data were analysed in five stages. First, descriptive statistics (mean, median, minimum, maximum, and standard deviation) were computed for each measurement occasion. Second, the Shapiro-Wilk test was used to assess normality, given that the per-school sample was fewer than fifty cases; because the distributions departed from normality, the non-parametric Wilcoxon signed-rank test was used to corroborate the pre-post comparison. Third, Interrupted Time Series analysis was conducted using a segmented regression model of the form

$$Y_t = \beta_0 + \beta_1(\text{Time}) + \beta_2(\text{Intervention}) + \beta_3(\text{TimeAfter}) + \varepsilon_t$$

where β_0 is the baseline intercept, β_1 is the pre-intervention trend, β_2 is the immediate change in level following the intervention, and β_3 is the change in trend after the intervention. Fourth, the effect size was computed using Cohen's d (with the pooled standard

deviation), interpreted as small (0.20), medium (0.50), or large (0.80). Fifth, the normalized gain (N-gain) was computed as $g = (\text{posttest} - \text{pretest}) / (\text{maximum score} - \text{pretest})$, with $g > 0.70$ categorized as high, 0.30–0.70 as medium, and < 0.30 as low.

4. Result

4.1 Implementation of the Learning Process

The study was organised around six measurement occasions, comprising three pre-intervention and three post-intervention assessments. During the intervention phase, multiplication instruction followed a structured sequence that integrated conceptual exploration, mathematics card play, group discussion, reflection, and conceptual generalisation. Rather than presenting multiplication as a fixed procedure, the teacher introduced numerical relationships through card-based challenges that required students to identify patterns, compare possible strategies, and explain their reasoning.

Classroom observations showed consistently high student involvement throughout the learning activities. Most students responded enthusiastically to the card game, participated actively in group tasks, and interacted with peers when constructing solutions. The activity generated multiple solution strategies, indicating that students were not merely reproducing teacher-demonstrated procedures. Instead, they explored numerical relationships, tested alternative approaches, and negotiated the validity of their strategies through discussion.

The teacher primarily acted as a facilitator, providing prompts, clarification, and scaffolding when students encountered difficulty without immediately supplying answers. This instructional pattern gradually shifted responsibility for solving problems from the teacher to the students. The card game therefore functioned not merely as an entertaining learning medium but as a structured problem space that encouraged mathematical reasoning, strategic independence, and collaborative knowledge construction.

A particularly important implementation finding was the relationship among play, discussion, and reflection. Card-based exploration generated initial strategies, group interaction exposed students to alternative approaches, and reflection required them to convert intuitive actions into explicit mathematical explanations. This sequence enabled students to move

from manipulating numbers to articulating the principles underlying multiplication. The observed engagement was therefore both behavioural and cognitive: students participated actively while also examining patterns, defending solutions, and refining their understanding.

4.2 Descriptive Statistics of Mathematical Problem-Solving Ability

The descriptive results reveal a clear separation between the pre-intervention and post-intervention phases. As shown in Table 4, mean problem-solving scores remained low and relatively stable during the three pretests, increasing only from 21.34 to 23.95. The total change across the baseline phase was 2.61 points, indicating that repeated testing alone produced little improvement before the intervention.

By contrast, the mean increased by 50.59 points between Pretest 3 and Posttest 1, from 23.95 to 74.54. Scores then continued to improve to 77.11 at Posttest 2 and 81.52 at Posttest 3. The average score across the three pretests was 22.60, compared with 77.72 across the three posttests, representing an overall phase difference of 55.12 points. This pattern indicates that the principal improvement occurred immediately after the intervention, followed by further consolidation rather than regression toward baseline performance.

Table 4. Descriptive Statistics of Mathematical Problem-Solving Ability Across Six Measurement Occasions (0–100; $n = 132$)

Occasion	n	Mean	SD	Mdn	Min	Max	Change in Mean
Pretest 1	132	21.34	18.93	17	0	80	—
Pretest 2	132	22.52	20.18	17	0	80	+1.18
Pretest 3	132	23.95	20.46	20	0	90	+1.43
Posttest 1	132	74.54	23.14	84	4	100	+50.59
Posttest 2	132	77.11	21.06	84	10	100	+2.57
Posttest 3	132	81.52	16.73	84	27	100	+4.41

Note. Mdn = median; SD = standard deviation. Change in mean represents the difference from the immediately preceding measurement.

Table 4 also reveals changes in the distribution of achievement. During the baseline phase, the median remained between 17 and 20, confirming that low performance characterised the typical student rather than only a small subgroup. Following the intervention, the median rose sharply to 84 and remained at that level

across all three posttests. The simultaneous increase in the mean and median demonstrates that the improvement occurred across the broader cohort rather than being driven solely by a few high-performing students.

The maximum score reached 100 at each post-intervention measurement, showing that some students achieved mastery immediately after the intervention. More importantly, the minimum score increased progressively from 4 at Posttest 1 to 10 at Posttest 2 and 27 at Posttest 3. This upward movement in the lower end of the distribution indicates that students who initially benefited less from the intervention continued to improve during subsequent measurements.

The standard deviation initially increased from 20.46 at Pretest 3 to 23.14 at Posttest 1. This temporary widening suggests that students responded to the intervention at different rates: some achieved high scores immediately, whereas others required additional exposure and practice. However, the standard deviation subsequently declined to 21.06 and then to 16.73. The narrowing distribution, combined with the rising minimum score, indicates increasing convergence in achievement and suggests that later improvement was particularly evident among lower-performing students.

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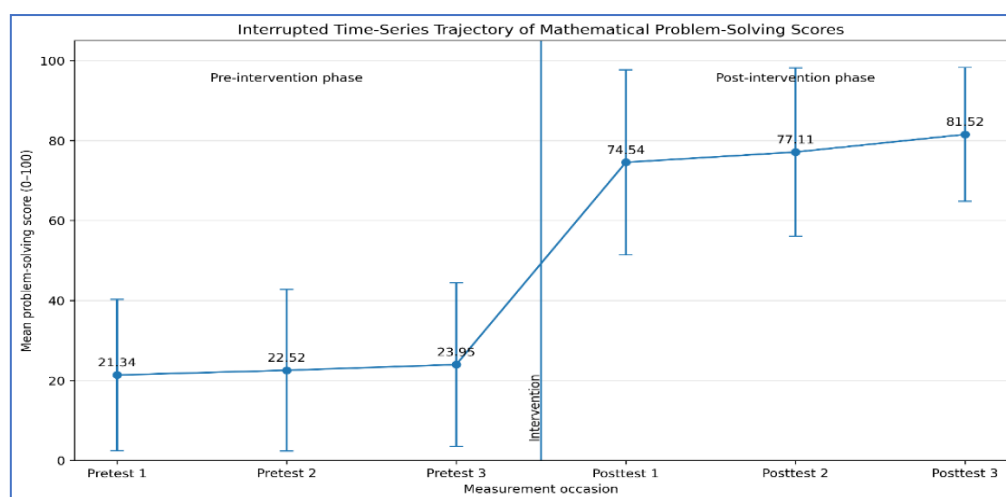


Figure 1. Interrupted Time-Series Trajectory of Mathematical Problem-Solving Scores

Note. Points represent cohort means, and error bars represent one standard deviation. The vertical line indicates the transition between the pre-intervention and post-intervention phases.

Overall, the descriptive evidence identifies four central findings. First, students' initial performance was low and stable. Second, the intervention was followed by an immediate and substantial increase in problem-solving scores. Third, achievement continued to improve across the post-intervention assessments. Fourth, the reduction in score dispersion and the progressive rise in minimum scores indicate that improvement increasingly extended to lower-performing students. Together, these patterns suggest that the intervention was associated not only with higher average achievement but also with a more equitable distribution of mathematical problem-solving performance across the cohort.

Figure 1 reinforces this interpretation by displaying a near-horizontal baseline followed by an abrupt discontinuity at the intervention point. The three pretest means form a stable trajectory, whereas the transition from Pretest 3 to Posttest 1 produces a pronounced upward shift. The increase is sufficiently large that the one-standard-deviation intervals surrounding Pretest 3 and Posttest 1 do not overlap, demonstrating that the immediate post-intervention difference was substantially greater than the ordinary variation observed within either occasion.

The post-intervention trajectory continues upward, although less steeply than the initial increase. This pattern distinguishes two components of change: an immediate intervention-associated improvement and a subsequent period of progressive consolidation. The absence of a post-intervention decline indicates that the initial gain was maintained, while the continuing increase suggests that students became more proficient in applying the newly developed strategies across successive assessments.

4.3 Time-Series Trend Analysis

The most prominent feature of the time series was a clear structural shift between the pre-intervention and post-intervention phases. During the baseline phase, mean scores increased only slightly from 21.34 at Pretest 1 to 22.52 at Pretest 2 and 23.95 at Pretest 3. The total baseline increase was 2.61 points, equivalent to an average descriptive rise of approximately 1.31 points per measurement occasion. This near-flat trajectory indicates that students' mathematical problem-solving ability remained consistently low before the intervention, with no substantial evidence of spontaneous improvement across the repeated pretests.

A markedly different pattern emerged immediately after the intervention. The mean increased by 50.59 points between Pretest 3 and Posttest 1, rising from 23.95 to 74.54. Scores then continued to increase to 77.11 at Posttest 2 and 81.52 at Posttest 3. The post-intervention phase therefore exhibited both an immediate change in performance level and a continuing positive trajectory. The average increase between successive posttests was approximately 3.49 points, indicating that the initial improvement was not followed by stagnation or decline. Instead, students appeared to consolidate and progressively extend the problem-solving skills developed during the intervention.

The stability of the baseline is particularly important for interpreting this pattern. Repeated exposure to the assessment before the intervention resulted in only minimal score changes, whereas the introduction of the GASING-based mathematics card game coincided with a sudden and substantial increase. Within the observed series, this discontinuity makes ordinary baseline drift or repeated-testing effects less plausible as the principal explanation for the improvement. Although the trend analysis alone does not establish causality, the timing, magnitude, and persistence of the change provide strong descriptive evidence of an intervention-related improvement.

Figure 2 further clarifies how the score distribution changed across the six measurement occasions. Before the intervention, both the means and medians remained concentrated near the lower end of the scale. The pretest medians ranged from 17 to 20, while the corresponding means ranged from 21.34 to 23.95. This pattern shows that low achievement characterised the majority of students rather than only a small group of low performers.

After the intervention, the median increased sharply to 84 and remained unchanged across all three posttests. The posttest means were initially lower than the median, indicating that a smaller group of students with comparatively low scores continued to pull the average downward. However, this lower-performing tail contracted over time. The minimum score increased from 4 at Posttest 1 to 10 at Posttest 2 and 27 at Posttest 3, while the standard deviation declined from 23.14 to 16.73. These changes suggest that later improvement was increasingly concentrated among students who had initially responded less strongly to the intervention.

The maximum score reached 100 at every post-intervention occasion, showing that some students attained the highest possible level immediately after the treatment. More importantly, the simultaneous rise in the minimum score and reduction in dispersion indicate that the intervention was not beneficial only to high-achieving students. The distribution gradually became more concentrated at the upper end of the scale, reflecting both stronger overall performance and greater consistency across the cohort.

Overall, the time-series pattern identifies three key findings. First, performance remained low and stable before the intervention. Second, the intervention was followed by an immediate and substantial upward shift. Third, scores continued to improve while variability decreased during the post-intervention phase. The combination of a stable baseline, a pronounced discontinuity, a sustained positive trend, and a contracting lower tail indicates that the improvement was both substantial and increasingly inclusive across students.

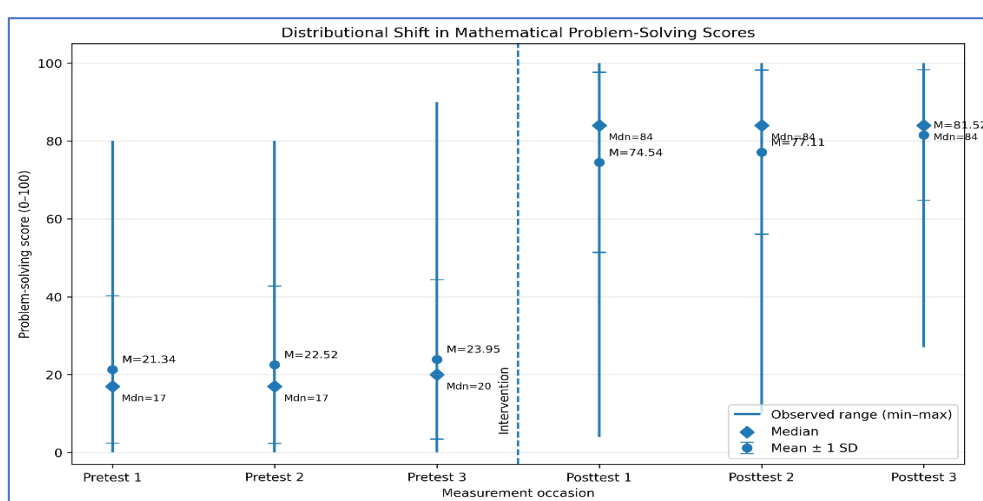


Figure 2. Distributional Shift in Mathematical Problem-Solving Scores Across Six Measurement Occasions

Note. Vertical ranges represent the observed minimum and maximum scores. Circular markers and error bars represent the mean and one standard deviation, respectively, while diamond markers indicate the median. The vertical dashed line marks the intervention point

4.4 Segmented Regression Analysis

Segmented regression was conducted to distinguish three components of the interrupted time series: the initial level of performance, the pre-intervention trend, and the changes in level and trend following the intervention. As shown in Table 5, the fitted model explained nearly all variation across the six measurement occasions, with ($R^2 = 0.9999$). This exceptionally close fit indicates that the segmented model accurately reproduced the observed trajectory. However, because the model was estimated from six aggregated time points, the (R^2) value should primarily be interpreted as evidence of descriptive fit rather than broad predictive capacity.

The estimated intercept was 19.99 ($p = .002$), representing the modelled starting level of students' mathematical problem-solving performance. The pre-intervention slope was 1.31 points per occasion. Although positive, this trend was not statistically significant ($p = .075$), confirming that performance remained broadly stable before the intervention. The small baseline slope is consistent with the descriptive pattern in which the three pretest means increased only marginally.

The principal result was the immediate level change associated with the intervention. The coefficient for the intervention was 46.83 ($SE = 0.95$), ($t = 49.07$), ($p < .001$), indicating a large upward displacement from the projected baseline trajectory. This coefficient was substantially larger than either the pre-intervention or post-intervention slope component, showing that the dominant effect was an immediate shift in performance rather than a gradual acceleration over time.

The post-intervention trend-change coefficient was positive at 2.18 points per occasion but narrowly missed the conventional threshold for statistical significance ($p = .055$). When combined with the baseline slope, the estimated post-intervention slope was approximately 3.49 points per occasion. Thus, the model identified two complementary patterns: a large and statistically reliable immediate improvement, followed by a smaller positive trajectory suggesting continued consolidation. The evidence for the immediate level change was strong, whereas the additional post-intervention acceleration should be interpreted more cautiously.

Table 5. Segmented Regression Results for Mathematical Problem-Solving Scores

Parameter	B	SE	t	p	Interpretation
Intercept (β_0)	19.99	0.82	24.40	.002	Estimated baseline starting level
Time/pre-intervention trend (β_1)	1.31	0.38	3.45	.075	Small, statistically non-significant baseline increase
Intervention/immediate level change (β_2)	46.83	0.95	49.07	< .001	Large and statistically significant upward shift
TimeAfter/trend change (β_3)	2.18	0.54	4.07	.055	Positive additional trend that narrowly missed significance

Note. ($R^2 = 0.9999$). The fitted model was [$Y = 19.99 + 1.31(\text{Time}) + 46.83(\text{Intervention}) + 2.18(\text{TimeAfter})$].

Overall, the segmented regression confirms that the observed score trajectory was characterised primarily by an abrupt intervention-associated level change. The small and statistically non-significant pre-intervention trend reduces the likelihood that the post-intervention increase merely continued an existing pattern. At the same time, the positive but borderline trend-change coefficient suggests that students may have continued strengthening their problem-solving performance after the initial gain, although this secondary pattern requires cautious interpretation.

4.5 Normalized Gain and Effect Size

Student-level scores were averaged across the three pretests and three posttests for 132 matched participants. The overall mean increased from 22.61 ($SD = 18.51$) before the intervention to 77.72 ($SD = 19.13$) after the intervention, producing an absolute increase of 55.11 points. The paired-samples analysis confirmed that this difference was statistically significant, ($t(131) = 29.40$), ($p < .001$). The Wilcoxon signed-rank test also yielded ($p < .001$), demonstrating that the improvement remained statistically reliable when evaluated using a non-parametric procedure.

The average normalized gain was ($g = 0.71$), placing the overall improvement in the high category. Cohen's ($d = 2.93$) indicated a very large effect. The agreement among the raw mean difference, normalized gain, parametric test, non-parametric confirmation, and effect-size estimate shows that the result was not merely statistically detectable. It represented a substantial educational improvement relative to students' initial performance and the available score range.

School-level results further demonstrate that the improvement was not confined to a single site. Every school recorded a large absolute increase, ranging from 39.64 to 60.91 points. Normalized gains ranged from 0.62 to 0.76, while Cohen's (d) values ranged from 1.86 to 3.91. Thus, all six schools experienced very large effects, although the relative efficiency of the gains varied across settings.

Table 6. School-Level Pre/Post Scores, Absolute Gains, Normalized Gains, and Effect Sizes

School	n	Pre M (SD)	Post M (SD)	Absolute Gain	g	Gain Category	d
SDN 5 Kaligung	11	48.18 (28.32)	87.82 (10.18)	39.64	0.76	High	1.86
SDN 2 Alasrejo	25	15.69 (11.14)	76.60 (18.98)	60.91	0.72	High	3.91
SDN Bakungan	20	19.73 (8.81)	76.90 (21.23)	57.17	0.71	High	3.52
SDN 4 Ketapang	19	19.02 (19.91)	78.67 (20.03)	59.65	0.74	High	2.99
SDN 2 Pakis	25	12.97 (6.59)	66.76 (20.21)	53.79	0.62	Moderate	3.58
SDN 1 Kalibaru Kulon	32	30.67 (18.89)	83.64 (15.40)	52.97	0.76	High	3.07
Overall	132	22.61 (18.51)	77.72 (19.13)	55.11	0.71	High	2.93

Note. Absolute gain = post-intervention mean minus pre-intervention mean. Normalized gains above 0.70 were classified as high, while values from 0.30 to 0.70 were classified as moderate.

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Figure 3 visualises the magnitude of the change within each school. The connecting lines show that every post-intervention mean was substantially higher than its corresponding pre-intervention mean. SDN 2 Alasrejo recorded the largest absolute increase at 60.91 points, followed by SDN 4 Ketapang at 59.65 points

and SDN Bakungan at 57.17 points. SDN 5 Kaligung showed the smallest absolute increase, although its post-intervention mean was the highest at 87.82. This apparently smaller gain reflects its comparatively high baseline score of 48.18 and therefore less available room for improvement.

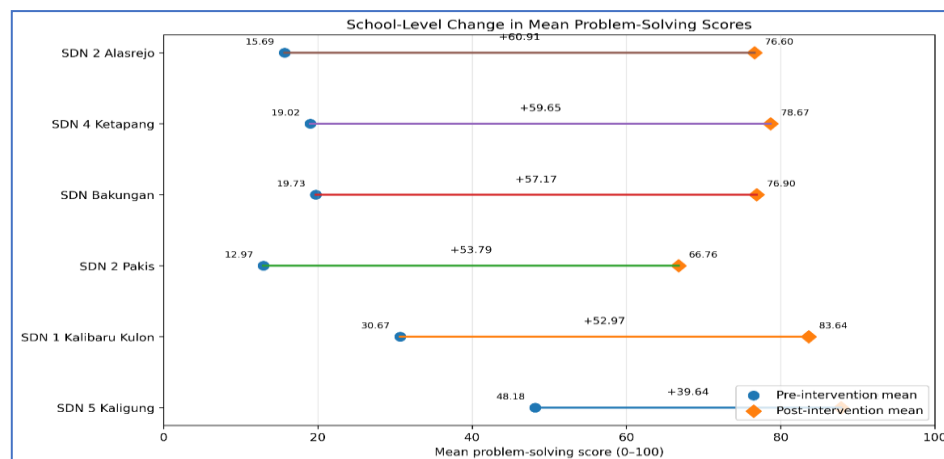


Figure 3. School-Level Change in Mean Mathematical Problem-Solving Scores

Note. Circles indicate pre-intervention means, diamonds indicate post-intervention means, and the values above the connecting lines represent absolute score gains.

Figure 4 presents normalized gains relative to the high-gain threshold of 0.70. SDN 5 Kaligung and SDN 1 Kalibaru Kulon achieved the highest normalized gains at 0.76, followed by SDN 4 Ketapang at 0.74. SDN 2 Alasrejo and SDN Bakungan also exceeded the high-category threshold, with gains of 0.72 and 0.71,

respectively. SDN 2 Pakis recorded the lowest normalized gain at 0.62, placing it in the moderate category. Nevertheless, its effect size remained very large at ($d = 3.58$), demonstrating that a lower normalized gain did not imply a weak improvement.

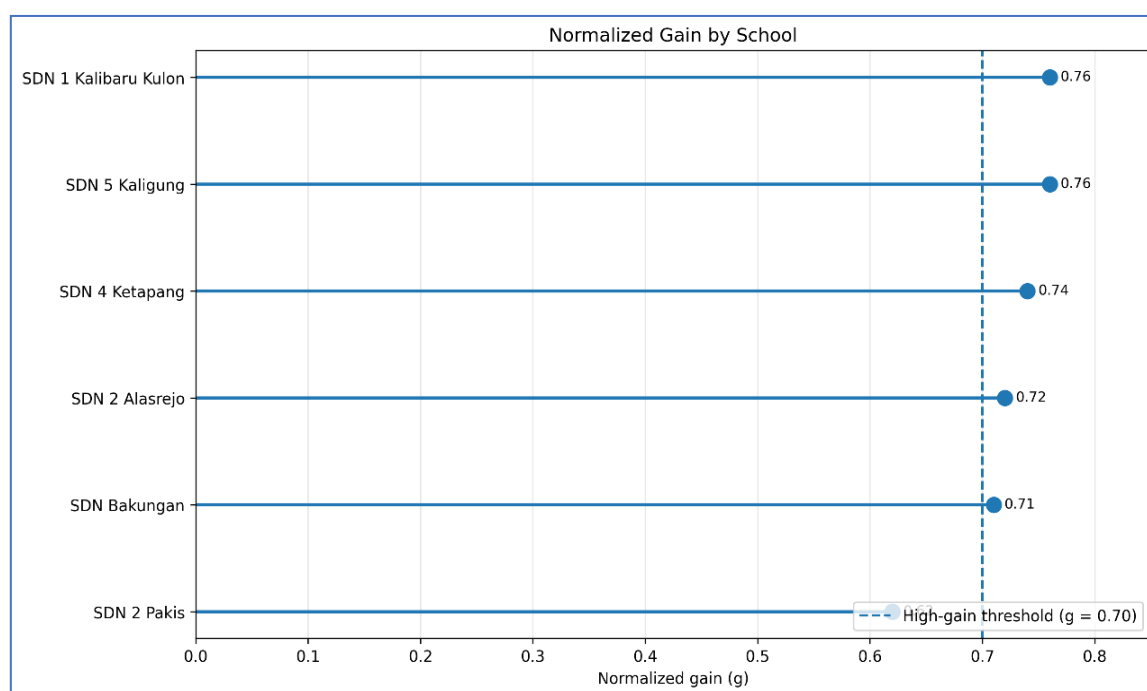


Figure 4. Normalized Gain by School Relative to the High-Gain Threshold

Note. The dashed vertical line represents the threshold for the high normalized-gain category, ($g = 0.70$).

The school-level patterns offer an important interpretation. Absolute gain, normalized gain, and effect size capture different dimensions of improvement and therefore do not produce identical rankings. SDN 2 Alasrejo achieved the largest absolute gain and effect size, whereas SDN 5 Kaligung and SDN 1 Kalibaru Kulon achieved the highest normalized gains. These differences reflect variation in initial scores, available improvement margins, and score dispersion. For example, SDN 5 Kaligung began with the highest pre-intervention mean, limiting its possible absolute increase, while SDN 2 Pakis began with the lowest mean but retained greater post-intervention variability.

Taken together, the results indicate that the intervention produced substantial improvement across schools with markedly different starting conditions. Five schools reached the high normalized-gain category, all six demonstrated very large effect sizes, and every school recorded a considerable increase in mean performance. The consistency of direction across

all sites strengthens the interpretation that the improvement was broadly reproducible rather than dependent on one unusually successful school.

4.6 Implementation Fidelity

Implementation-fidelity observations produced an overall mean of 90.5%, indicating that the instructional procedure was delivered with a very high degree of consistency. As shown in Table 7, all observed components exceeded 84%, although the results varied across specific aspects of implementation. The range between the highest and lowest scores was 11.1 percentage points, demonstrating strong overall delivery alongside some variation in the enactment of individual components.

The use of the mathematics card game achieved the highest fidelity score at 95.3%, exceeding the overall mean by 4.8 percentage points. This finding indicates that the central instructional medium was used consistently and in accordance with the planned procedure. Teacher activity also demonstrated high

fidelity at 92.4%, suggesting that the teacher effectively maintained the intended sequence of facilitation, questioning, guidance, and scaffolding.

Student activity reached 90.8%, closely matching the overall mean. This result shows that students generally participated as expected in card manipulation, pattern exploration, problem-solving, and classroom interaction. The close alignment between teacher activity and student activity also indicates that the instructional procedure was not implemented only at the level of teacher delivery; it was accompanied by substantial learner participation.

Collaboration obtained a score of 89.7%, only 0.8 percentage points below the overall mean. Although slightly lower than teacher and student activity, this score indicates that group interaction, strategy comparison, and peer communication were implemented consistently across the observed sessions.

Nevertheless, the difference suggests that collaborative participation may not have been equally distributed among all students or groups.

Reflection received the lowest score at 84.2%, falling 6.3 percentage points below the overall mean. Although this result remained high in absolute terms, it identifies reflection as the least consistently enacted component. Compared with the highly visible activities of playing, responding, and collaborating, reflective explanation required students to verbalise their reasoning, evaluate strategies, and connect game-based experiences with formal multiplication concepts. The lower score therefore suggests that the transition from active participation to explicit mathematical articulation was the most demanding part of the instructional sequence.

Table 7. Implementation-Fidelity Results Across Instructional Components

Rank	Aspect	Percentage (%)	Difference from Overall Mean
1	Use of the game	95.3	+4.8
2	Teacher activity	92.4	+1.9
3	Student activity	90.8	+0.3
4	Collaboration	89.7	-0.8
5	Reflection	84.2	-6.3
	Overall mean	90.5	—

Note. Difference values represent percentage-point deviations from the overall implementation-fidelity mean.

Figure 5 illustrates the fidelity profile across the five instructional components. Three aspects use of the game, teacher activity, and student activity were positioned above the overall mean. Collaboration remained close to the mean, whereas reflection was

visibly lower than the other components. The resulting profile indicates that the intervention was implemented most consistently in its procedural and participatory dimensions, while its reflective dimension showed comparatively greater implementation difficulty.

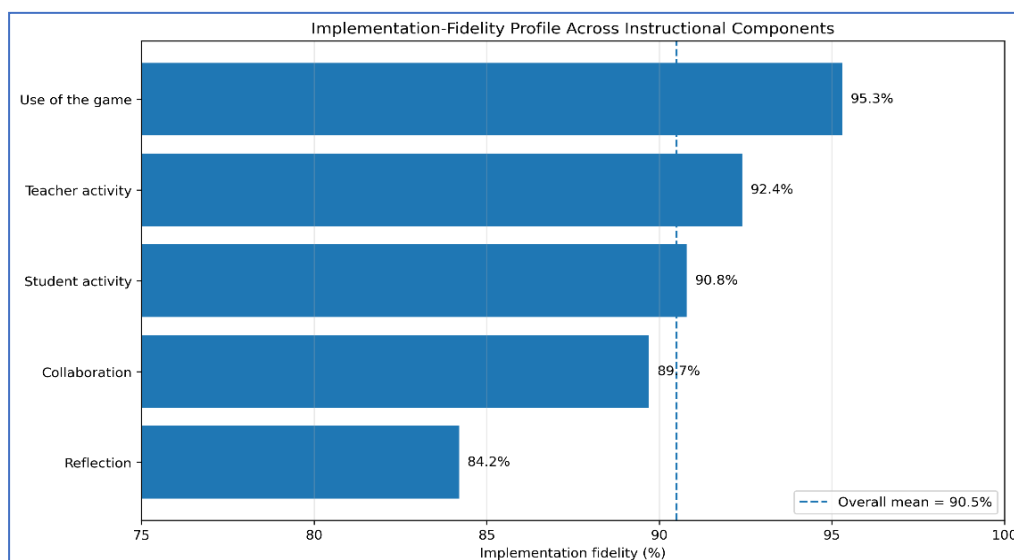


Figure 5. Implementation-Fidelity Profile Across Instructional Components

Note. The dashed vertical line represents the overall implementation-fidelity mean of 90.5%.

Overall, the findings confirm that the intervention was implemented substantially as designed. The high fidelity of the game, teacher activity, and student participation strengthens confidence that the observed learning outcomes were generated under conditions closely aligned with the intended instructional model. At the same time, the comparatively lower reflection score identifies a specific area for instructional refinement. More structured reflection prompts, individual explanation tasks, strategy-comparison sheets, or brief written mathematical justifications may help ensure that students consistently transform their game-based experiences into explicit conceptual understanding.

5. Discussion

The principal finding of this study is that implementing pedagogical deep learning through the GASING method supported by a mathematics card game was followed by a substantial and sustained improvement in fourth-grade students' mathematical problem-solving ability. The three pre-intervention means remained comparatively stable at 21.34, 22.52, and 23.95, whereas the three post-intervention means increased markedly to 74.54, 77.11, and 81.52. Segmented regression identified a large and statistically significant immediate level change following the intervention, ($\beta_2 = 46.83$), ($p < .001$), while the positive post-intervention trend, ($\beta_3 = 2.18$), ($p = .055$), suggested continued consolidation without reaching the conventional .05 significance threshold. Student-level analysis produced a high normalized gain of 0.71 and a very large effect size of ($d = 2.93$), with both the paired comparison and Wilcoxon signed-rank confirmation indicating statistically significant improvement. Gains also appeared across all six schools, with school-level normalized gains ranging from 0.62 to 0.76. These

converging results indicate that the observed improvement was not limited to one measurement occasion or a single school, although the absence of a comparison group requires the change to be interpreted as strongly associated with, rather than conclusively caused by, the intervention.

5.1 Deep Learning and the Construction of Mathematical Understanding

The improvement can first be understood through the conceptual orientation of the intervention. Instead of receiving multiplication procedures as finished rules, students manipulated number cards, explored relationships, identified recurring patterns, articulated strategies, and reflected on how their solutions were produced. Such activities encouraged learners to move beyond answer production toward an understanding of why multiplication procedures work. Research on meaningful mathematics learning similarly demonstrates that students develop stronger and more transferable knowledge when instruction directs their attention to numerical structures and relationships rather than isolated computations (Björklund et al., 2021; Sterner et al., 2020). The cards therefore served not merely as physical materials but as representational resources through which students could connect concrete numerical arrangements with multiplication concepts.

This interpretation is also consistent with research on representational sequencing. Concrete materials are not automatically effective simply because students can manipulate them; their instructional value depends on whether learners are helped to recognize the mathematical structures represented by the objects and to connect those structures with pictorial and symbolic forms (Fyfe & Nathan, 2019; Kokkonen & Schalk, 2021). In the present intervention, the progression from card manipulation to pattern identification, group

explanation, reflection, and symbolic generalization created explicit links among action, speech, visualization, and notation.

This sequence may have reduced the conceptual distance between students' immediate experiences and the abstract multiplication representations they were expected to use. Studies of early mathematics interventions similarly show that collective reasoning about multiple representations can strengthen number understanding when the relationships among the representations are made visible and discussable (Sterner et al., 2020). Formative opportunities to explain and reconsider strategies may also support mathematical reasoning by strengthening students' confidence in participating in problem-solving processes (Smit et al., 2023).

5.2 The Contribution of GASING Sequencing to Problem-Solving Development

The GASING method appears to have contributed to the findings through its gradual, pattern-oriented, and success-building instructional sequence. Students began with accessible card-based activities before progressing toward increasingly formal representations of multiplication. This concrete to semi-concrete to abstract progression enabled them to examine relationships among quantities before relying on conventional notation. Rather than memorizing multiplication facts as disconnected verbal sequences, students were encouraged to notice number pairs, repeated groups, and regularities that could be converted into efficient solution strategies. Such structural awareness is central to mathematical problem solving because it enables learners to identify relationships within a problem instead of approaching every item as an unfamiliar computational task.

The pattern-discovery component also appears to have supported the four problem-solving processes assessed in this study. When students formed number combinations, they had to interpret the information represented by the cards. When they compared possible multiplication strategies, they planned alternative solution procedures. When they used the chosen strategy, they executed the mathematical operation. Finally, group discussion and reflection provided opportunities to reconsider the reasonableness of the result. Game-based number-learning environments have similarly been found to strengthen adaptive number knowledge when students are required to recognize numerical structures and select strategies rather than merely produce rapid answers (Brezovszky et al., 2019).

Research on number-sense games also indicates that representations, feedback, and opportunities to explain patterns can connect informal numerical intuitions with early algebraic reasoning (Hulse et al., 2019). Furthermore, combining modelling, collaboration, and game activity can improve

mathematics performance when students are given opportunities to observe relationships, test ideas, and discuss their reasoning with peers. The contribution of GASING in this study should therefore be understood not as the effect of enjoyable practice alone but as the effect of structured enjoyment directed toward progressively deeper mathematical reasoning.

5.3 The Mathematics Card Game as a Space for Numeracy Discourse

A second explanation concerns the social and discursive role of the mathematics card game. Classroom observations showed that students were highly engaged in manipulating cards, negotiating strategies, comparing answers, and working collaboratively. Rather than serving merely as entertainment, the game created a structured space for students to articulate mathematical ideas, consider alternative strategies, justify decisions, and revise their reasoning (Hayden et al., 2024; Higgins & Parsons, 2021; Yang et al., 2023). Numeracy was therefore enacted as classroom discourse through prompts, talk moves, and collaborative reasoning (Hayden et al., 2024; Higgins & Parsons, 2021). By connecting physical arrangements, oral explanations, numerical patterns, written procedures, and symbolic conclusions, the activity strengthened mathematical vocabulary and communicative fluency (Hayden et al., 2024; Higgins & Parsons, 2021; Yang et al., 2023). Such dialogic interaction is associated with improved mathematical talk, learner ownership, and academic outcomes, while code-switching and translanguaging can extend participation in multilingual classrooms (Higgins & Parsons, 2021; Yamaguchi, 2025; Yang et al., 2023).

This finding supports evidence that game-based learning can promote cognitive and affective outcomes in mathematics. Meta-analytic research indicates positive effects on achievement when games are closely aligned with curricular objectives and provide meaningful mathematical activity (Tokac et al., 2019). Games may also enhance motivation, cognition, and participation by offering clear goals, manageable challenges, immediate consequences, and repeated opportunities for success (Sailer & Homner, 2020). Mathematics-focused reviews similarly associate game-based learning with improved knowledge, skills, attitudes, motivation, interest, and engagement, although outcomes vary across designs and learner groups (Hui & Mahmud, 2023; Vankúš, 2021). Recent reviews further stress that teachers must maintain an explicit connection between gameplay and the intended mathematical concept to prevent play from becoming detached from learning (Debrenti, 2024; Hidayat et al., 2024).

Teacher facilitation was crucial to sustaining this connection. The teacher explained the rules, supported students experiencing difficulty, encouraged strategy comparison, and guided the final generalisation of multiplication concepts. Learners benefit most from

scaffolding that clarifies the mathematical demands of a game without removing their responsibility for solving problems (Sun et al., 2021). Thus, the intervention's success cannot be attributed to the cards alone but to the interaction among game design, GASING sequencing, teacher scaffolding, peer communication, reflection, and explicit mathematical generalization.

5.4 Understanding the Magnitude and Distribution of Improvement

The magnitude of the observed improvement warrants both recognition and caution. The normalized gain of 0.71 indicates that students achieved a large proportion of the improvement available between their baseline performance and the maximum score. The effect size of ($d = 2.93$) is also substantially larger than the small to moderate average effects commonly reported in broader syntheses of game-based and digital STEM learning (Gui et al., 2023; Kacmaz & Dubé, 2022; Wang et al., 2022). One plausible interpretation is that the intervention combined several mutually reinforcing components rather than relying on a single game mechanism. Students received concrete representations, graded practice, peer interaction, pattern exploration, teacher scaffolding, reflection, and repeated opportunities to apply problem-solving strategies. The initially low baseline scores may also have provided substantial room for improvement, while close alignment between the instructional activities and the assessment indicators may have strengthened the measured gains.

Nevertheless, this unusually large effect should not be interpreted as evidence that the intervention would invariably produce the same magnitude of improvement in other settings. Research syntheses demonstrate that game-based outcomes vary according to pedagogical approach, mathematical content, game mechanics, teacher support, duration, learner characteristics, and the comparison condition (Gui et al., 2023; Hussein et al., 2022; Kacmaz & Dubé, 2022). Digital and non-digital games may also generate different patterns of cognitive load, feedback, social interaction, and motivation, making direct comparisons difficult (Talan et al., 2020). Moreover, studies of educational mathematics games have sometimes produced mixed cognitive and non-cognitive outcomes, illustrating that engagement does not automatically guarantee conceptual improvement (Vanbecelaere et al., 2020). The present effect size may therefore represent the combined influence of the instructional intervention, the low initial scores, repeated exposure to similarly structured assessments, and context-specific implementation conditions.

The time-series evidence also requires proportionate interpretation. Repeated pre-intervention and post-intervention measurements provide a stronger description of temporal change than a single pretest and posttest because they allow baseline stability and post-

intervention development to be observed. However, the analysis was based on only six measurement occasions and fitted four regression parameters to the aggregate occasion means. The near-perfect (R^2) should consequently be interpreted as evidence of close model fit within these six observations, not as proof that virtually all variation in students' mathematical performance was explained by the intervention. Interrupted time-series analyses are generally strengthened by a larger number of observations, examination of autocorrelation and seasonality, and the inclusion of a concurrent comparison series when feasible (Schaffer et al., 2021). The immediate level change is therefore the strongest component of the segmented-regression evidence, whereas the non-significant pre-intervention and post-intervention trend coefficients should be discussed descriptively rather than treated as confirmed trend effects.

5.5 Implementation Fidelity and Potential Inclusiveness

The implementation-fidelity mean of 90.5% provides additional support for the internal coherence of the findings. Teacher activity, student activity, game use, and collaboration were implemented at high levels, suggesting that the instructional procedure was generally enacted as planned. This evidence is important because a strong intervention outcome cannot be interpreted meaningfully without determining whether the intended instructional components were actually delivered. The high game-use score of 95.3% indicates that the card activity occupied a clear role in the intervention, while the collaboration score of 89.7% suggests that peer interaction was consistently incorporated. Reflection obtained the lowest component score at 84.2%, although it still remained within the very-good category. This comparatively lower result indicates that reflective explanation may require further instructional attention because students can become absorbed in completing game challenges without fully verbalizing the principles underlying their strategies.

The narrowing of score dispersion by the third post-intervention measurement also deserves attention. The standard deviation declined from 23.14 at the first posttest to 16.73 at the third, while the minimum score increased from 4 to 27. This pattern suggests that continued participation may have benefited not only initially successful students but also learners who began the post-intervention phase with comparatively weak performance. The interpretation remains tentative because changes in subgroup achievement were not formally tested. Nevertheless, the collaborative, concrete, and comparatively low-risk learning environment may have enabled struggling students to attempt strategies without experiencing the evaluative pressure commonly associated with conventional mathematics exercises. This possibility is educationally important because mathematics anxiety

is consistently associated with lower achievement, reduced self-esteem, avoidance, and reluctance to participate in problem-solving activities (Barroso et al., 2021; Su & Guo, 2022; Wei & Yi, 2025).

Repeated experiences of confusion or failure may intensify these effects, making emotional safety essential to effective mathematics instruction. However, enjoyment must remain connected to meaningful cognitive engagement rather than becoming an end in itself. Cognitive-behavioural interventions and related psychosocial support can reduce mathematics anxiety while improving engagement and achievement (Ojo et al., 2023; Wei & Yi, 2025). At the school level, successful adoption requires teacher preparation, collaborative planning, and sustained facilitation because educational innovations often weaken when instructional principles are transferred without sufficient implementation support (Rösken-Winter et al., 2021; Hawes et al., 2021; Mandhasari, 2026). Approaches that strengthen teacher knowledge, provide reflective support, encourage collaborative learning, and promote a growth mindset have produced positive outcomes for both teachers and students (Hawes et al., 2021; Mandhasari, 2026).

5.6 Research Gaps, Novelty, and Implications

The present study addresses several gaps in mathematics game-based learning. First, much of the international literature has concentrated on digital games, whereas the potential of low-cost physical card games to support mathematical discourse, face-to-face collaboration, and representational movement remains less extensively investigated (Bertram, 2020; Hussein et al., 2022; Talan et al., 2020). Second, many previous studies have examined achievement or motivation as separate outcomes without explaining how gameplay, representational sequencing, teacher scaffolding, reflection, and problem-solving processes operate as an integrated pedagogical system. Third, mathematics games are frequently designed around drill, factual recall, and behavioural reinforcement even when their stated objectives include conceptual understanding (Kacmaz & Dubé, 2022). Fourth, one-time pretest-posttest comparisons continue to dominate classroom intervention studies, limiting the ability to distinguish an immediate score difference from a developing pattern of change.

The novelty of this study lies in addressing these gaps through the integration of pedagogical deep learning, GASING sequencing, a physical mathematics card game, collaborative numeracy discourse, and Polya-oriented problem-solving assessment across six elementary schools. The cards were not used solely to increase enjoyment or repetitive fluency. They were embedded within a sequence of orientation, concrete exploration, pattern discovery, group discussion, reflection, and conceptual generalization. The study also combined repeated outcome measurements with

implementation-fidelity observations, thereby linking changes in performance with evidence about how the intervention was enacted. To the extent supported by the available literature, this integrated configuration remains underrepresented in elementary mathematics research. The contribution is therefore both pedagogical and methodological: it demonstrates how a culturally recognizable, accessible instructional method can be redesigned as a deep-learning environment and evaluated through more than a single before-and-after measurement.

These findings carry practical implications for teachers, schools, and curriculum developers. Teachers can employ carefully designed card games to elicit explanations, expose alternative strategies, and make numerical structures visible, but they should explicitly connect every game action to the intended mathematical concept. Reflection should be strengthened by asking students to explain why a strategy works, compare efficient and inefficient solutions, identify errors, and represent the same relationship in different forms. Schools can incorporate similar games into numeracy programmes because physical cards require limited technological infrastructure and can be adapted to varied classroom conditions. However, successful scaling requires teacher development in scaffolding, formative questioning, mathematical representation, and classroom facilitation rather than the distribution of game materials alone. For curriculum developers, the intervention illustrates how deep learning can be operationalized through concrete tasks that integrate conceptual understanding, communication, collaboration, critical reasoning, and reflection. These implications support the Merdeka Curriculum's movement away from content transmission toward meaningful, learner-centred, and competency-oriented mathematics instruction.

5.7 Study Boundaries and Future Research

Several boundaries should guide the interpretation and extension of these findings. The absence of a control group prevents firm exclusion of maturation, repeated-testing effects, teacher expectancy, and other concurrent influences. The intervention addressed only multiplication, was implemented within one regency, and was observed for a relatively short period. The use of three measurements in each phase improves on a single pretest-posttest design but remains insufficient for comprehensive time-series diagnostics. Future studies should therefore employ randomized or matched comparison groups, controlled interrupted time-series designs, and a larger number of pre-intervention and post-intervention observations. Delayed posttests should examine whether the gains persist after the novelty of the game has diminished.

The model should be further tested across a wider range of mathematical topics and in diverse rural, urban, multilingual, inclusive, and socioeconomic educational

contexts to assess its broader applicability and effectiveness. Comparative studies could evaluate physical, digital, and blended games under equivalent content and instructional time. Future research should also examine whether conceptual understanding, discourse, self-efficacy, engagement, cognitive load, mathematics anxiety, and peer support mediate gains in problem-solving achievement. Finally, process-oriented research using video observation, student interviews, written strategy analysis, and teacher reflection should identify which GASING and game components produce durable learning and which require redesign for different learners and educational contexts.

6. Conclusions

This study demonstrates that a structured sequence of play, reflection, and problem solving can transform elementary multiplication learning from procedural practice into an active process of conceptual reasoning. The integration of pedagogical deep learning, GASING sequencing, and a physical mathematics card game produced a stable pre-intervention baseline followed by a substantial and sustained improvement in students' mathematical problem-solving performance. Mean scores increased from 22.61 before the intervention to 77.72 afterward, segmented regression identified a large immediate level change ($\beta_2 = 46.83$, $p < .001$), the normalized gain was high ($g = 0.71$), and the effect size was very large ($d = 2.93$). Improvements occurred across all six participating schools, while the implementation-fidelity score of 90.5% confirmed that the instructional model was delivered consistently.

The study's novelty lies in combining concrete card manipulation, pattern discovery, collaborative strategy construction, mathematical explanation, reflection, and symbolic generalisation within one accessible instructional architecture, while evaluating change through repeated measurements rather than a single pretest-posttest comparison. These findings indicate that low-cost physical games can function as intellectually demanding learning environments when gameplay is explicitly connected to mathematical reasoning and teacher scaffolding. For teachers and schools, the model offers a practical approach to strengthening numeracy, participation, and mathematical communication; for curriculum developers, it illustrates how meaningful, mindful, and joyful learning can be operationalised within learner-centred mathematics instruction. Future studies should examine the model through controlled or randomized designs, longer intervention periods, and more measurement occasions, while extending its application to fractions, geometry, measurement, algebraic reasoning, mathematical creativity, anxiety, motivation, and long-term retention across diverse educational contexts.

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